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A further explanatory note on the Riemann hypothesis (i) – Peter Braun

'For there is good news yet to hear and fine things to be seen,
Before we go to Paradise by way of Kensal Green'
-G.K. Chesterton

Notes [1] and [2] represent a gathering together of information and ideas to explain why the Riemann hypothesis (RH) is unprovable. The idea in the form of presentation being developed over a number of drafts is that the processing of half baked ideas is a time consuming process and any confusion and misconceptions will be helped by discussion and re-expression.

There are some reasons for considering unprovability as a possibility:

- The published insights from mathematical physics
- The division between for and against mathematicians – Littlewood/Hardy
- The volume of equivalent propositions which suggest RH is something mathematics bounces off
- Unprovability implies simple zeros and this is the case so far
- All calculated zeros lie on the line
- The problem has remained unsolved for 150 years
- The author spent 40 years trying to prove it by elementary methods

An early passage in [2] includes

'The principal idea in this note is that if a theorem has a finite proof then the propositions which make up the essence of the proof will only ever generate a finite number of essentially different propositions.'

We call this the finite proof, finite theorem assertion (FPFTA).

Another way of expressing FPFTA is to say - what we get out of a piece of mathematics in a proof, in some global accounting, must equal the amount we put in and this must be a finite amount (a variation on Newton's third law). The 'value added' extras are in the value of the explanation. Proofs are seen as explanations which follow from collection of assumptions and the stronger proofs are those which we are able to relate back to our direct experience. For example the uniqueness of factorisation in the natural numbers may be discovered with a pencil and paper looking at the factorisation of 1, 2, 3, 4, 5, 6 …..

The development of the explanation of this from some elementary assumptions introduces the notion of proof. The relationship of a proof to our experience determines the extent to which a proof is accepted. If we accept the explanation we call it a valid proof. It seems unlikely that we would ever expect more than this. To ask for more may be asking for a commodity which is non-existent within our understanding. We use the laws of classical logic to trust our decision making. The idea of quite easily stated theorems in number theory which may be unprovable has not been much in favour. But given the unique and difficult distribution of prime numbers, the possibility of getting sensible new results from unprovable statements does seem possible. These could include results which are still provable by more conventional methods.

A proof of unprovability is a different sort of proof but it is the explanatory function of proof which needs to be emphasised.

It has been suggested that RH would require a new axiom – possibly RH itself. It turns out that we need one less axiom, the axiom of infinity. Not because this axiom is a confounding factor but because it is not necessary and causes unnecessary confusion between mathematical mechanisms and logical mechanisms in the context of RH. We need to understand what we may assume and the contexts in which the assumptions are appropriate.

The question of counter example is discussed in the context of Fermat's last theorem (FLT).

The Wiles proof, which is finite, produces an unbounded number of propositions each of which is true.

Namely, $X^n+Y^n = Z^n$ has no non-trivial solution in integers for $n> 2$.

Here then with p(n) denoting this statement for the number n, the statement of FLT is essentially p(3)^p(4)^…..^p(n)^…. is true.

We should note from the finiteness discussions in the earlier drafts that this type of unbounded collection of propositions is merely using the existence of an unbounded number of natural numbers to artificially construct an unbounded sequence of true propositions. This is imposing a mathematical construction on logic but classical logic only applies in finite argument.

We need to distinguish between the proof of (FLT) and the consequences which flow from the proof. The finite proof can only consist of a finite number of distinct propositions otherwise the proof could never be read and understood. Having established a proof we may then generate the unbounded propositions in the manner described. But there is nothing unbounded about the reasoning. This is also true for any theorem q(n) which has been proved using the standard induction principle from Peano arithmetic.

In the preceding drafts the suggestion is made to disallow this usage. The use of 'for all' clouds over what has actually been proven. The nicety of having an axiom of infinity so we feel more comfortable that a proof is on a more solid footing and covers a general case doesn't really produce any increased validity but provides a convenient way of stating theorems. The view is that the notion of unboundedness as exhibited in the pattern 1, 1+1, 1+1+1, … cannot be extended to describe an entity which has a valid existence. There is no extra truth in saying a theorem is true for all natural numbers.

It is important here to understand that the convention of avoiding theorem statements which have this form is not designed to provide an 'argument' that theorem such and such is unprovable. If we took this view we would have to throw out all theorems proved by mathematical induction and that would not be very popular. This convention is adopted merely to help unbundle the logic working within a theorem proof from the mathematical constructs within the proof.

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Note also at this point we have yet to address the notion of 'essentially different' propositions.

Without assuming a proof of FLT in its usually stated form about non-existence of solutions, we need to decide whether an arbitrary sequence of propositions p(3), p(4)… p(N) are somehow related or whether they are sufficiently different so that the possibility of a counter example can never be excluded. Wiles succeeded in finding a common theme in the propositions. This closed the door on bothering to look for a counter example $X^N + Y^N = Z^N$ for some $n > 2$.

After a proof of FLT we could say that p(3), p(4), .. p(n) … are all equivalent to each other (because they are all true) but we certainly cannot say that each p(n) is equivalent to FLT in the proof sense unless we take the trivia path of assuming the Wiles proof.

To date we do not have an abstract mathematical proof that (say) p(n) implies p(n+1) for FLT and outside of the unifying ideas of Wiles the interrelationships between the p(n) is fragmented.

Further explanation of FPFTA

Now let Q(N) denote the theorem $M_N(X) = O(X^{(K-1/2+\epsilon)})$ as $X\to\infty$.

What then is it about the proposition sequence,

$RH \equiv Q(1) \equiv Q(2) \equiv Q(3) \equiv Q(4) \ldots \equiv Q(N)$ ….., that allows us to say RH is unprovable?

Suppose that a finite proof of RH had been given.

We note that for $N \geq 1$, $Q(N) \rightarrow Q(N+1)$ is a true theorem but $Q(N+1) \rightarrow Q(N)$ is non–trivial and cannot be a general theorem because we can construct a sequence for which the result does not follow.

Further we are able to explain that a logical hierarchy exists where we have a sensible meaning for $Q(1) > Q(2) > \ldots > Q(N) > \ldots$ in the smoothing which may occur in the consideration of higher summation functions.

For $A_1(X), A_2(X) \ldots A_n(X)$.. where $A_n(X) = \sum A_{(N-1)}(X)$ the larger the value of n, the less likely the implication $A_n(X) \rightarrow A_1(X)$ is to be true for sequences. In other words, more is being asked of Q(N) in $Q(N) \rightarrow Q(1)$ than is being asked of Q(N-1) in $Q(N-1) \rightarrow Q(1)$.

Thus a proof of RH would have within its logical structure the generation of an unbounded number of hierarchically different propositions.

In a sense the proof of RH would generate the natural numbers 1,2,3…..

i.e. an unbounded number of distinct patterns (theorems).

The problem: prove the natural numbers exist – would not be contemplated as a decidable proposition but this is what is generated by the existence of RH as a question, the generation of an unbounded number of essentially different entities.

A counter example would need to generate an unbounded number of equivalent propositions with an inbuilt hierarchical order and this would require an unbounded number of proofs of propositions. Since no finite counter example is possible it will not be possible to find a zero off the line $\sigma = 1/2$ through computation, as this would essentially be a finite proof.

The excluded middle

We know we need to keep open the possibility that a stated theorem may be true, false, unprovable or not well defined. An immediate worry in the intuitionist excluded middle EM notion is the validity of theorems which are proved by obtaining a contradiction. It is not the intention here to discuss this problem as it is not a critical issue in a problem like RH, but a certain orientation is required (with RH we do not get a contradiction following from an assumption, but a collection of conditions which cannot be met).

In draft 1 we asserted that if RH was unprovable we could not assume RH true or RH false and expect to get anything in the way of proof. This does seem to have an intuitionist leaning but in review it appears the EM notion is only used as a prop to take away absolute focus on the true/false universe. In other words, we do not use EM in order to get an explanation of unprovability. It may appear that that the argument $RH \equiv Q(1) \equiv Q(2) \equiv Q(3) \equiv Q(4) \ldots \equiv Q(N)$ .. is in some way assuming RH, in which case an objection may be that virtually anything may be shown to follow from a false assumption.

It is this objection we deal with here.

From the theory of RH and complex analysis there exists a least number α for which the statement $\zeta(s) \neq 0$ for $\sigma > \alpha$ is true.

This is the α of the quasi-Riemann as discussed by Braun and Zulauf [1]. And so we have α at our disposal quite independently of RH.

Then the unbounded equivalent theorems which follow from the quasi Riemann hypothesis may be used in the unprovability argument.

The quasi –Riemann hypothesis may be taken to be true for some α, and the unprovability argument is starting from a true theorem. i.e. even though α exists, the value of α in $[1/2, 1]$ is undecidable.

References

[1] A note on the Riemann hypothesis (i) https://www.peterbraun.com.au

[2]A note on the Riemann hypothesis (ii) https://www.peterbraun.com.au

[3] P.B. Braun – Topics in Number Theory. D.Phil thesis. University of Waikato 1979.

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