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A note on Goldbach's conjecture (GC)- Peter Braun

Every even number may be written as the sum of two odd primes.

An early passage in Braun [3] includes:

'The principal idea in this note is that if a theorem has a finite proof then the propositions which make up the essence of the proof will only ever generate a finite number of essentially different propositions.'

We call this the finite proof, finite theorem assertion (FPFTA).

A form of this principle is used to discuss the unprovability of the Riemann hypothesis and also the twin prime problem and a generalisation.

Suspicion about the provability of GC led the author to look for a general statement which could be used to argue unprovability. Having located a suitable generalisation there is very little left to do.

We assume that unity is included as a prime.

The general proposition, p(N) for $N \geq 2$, is that every multiple of N may be written as the sum of N odd primes.

The structure of the discussion is to show that p(2) implies p(3), p(4).....

Then to note that p(3), p(4) ... are essentially different propositions and conclude that p(2) is unprovable.

Hence, we will always find confirmation of the theorem in numerical investigations.

We assume the truth of p(2) and examine a general case p(N).

Since N = 1+1+ ....+1 (N times), 2N = 2+2+…+2 (N times) and 3N= 3+3+…+3 (N times), we have verified p(N) for the first few multiples of N.

Now assume we have the theorem true for 1N, 2N … (K-1)N.

If N is even

Then, KN = (K-1)N + N

$= p_1+p_2 + \ldots + p_{(N-2)} + (p_{(N-1)}+p_N +N)$

$= p_1+p_2 + \ldots + p_{(N-2)} + 2M$

$= p_1+p_2 + \ldots + p_{(N-2)} + (Q_1+Q_2)$

where p_1, p_2 … p_N, Q_1, Q_2 are all odd primes. Then KN is written as the sum of N odd primes.

If N is odd, we may assume K> 3 from earlier remarks.

Then KN = (K-2)N + 2N

$= p_1+p_2 + \ldots + p_{(N-2)} + \ldots +(p_{(N-1)}+p_N +2N)$

$= p_1+p_2 + \ldots + p_{(N-2)} + \ldots +(2M)$

$p_1+p_2 + \ldots + p_{(N-2)} + Q_1+Q_2$

where p_1, p_2 … p_N, Q_1, Q_2 are all odd primes.

Once again we have KN as the sum of N odd primes.

With p(3), p(4), p(5)…. essentially logically different propositions it follows that p(2) is unprovable and hence will be free from counter examples.

Discussion of a proposed counter- example

The author examined a suggested counter-example for the explanation of the Riemann hypothesis Braun [3] in this context.

Let q(n) be the theorem: $X^n + Y^n = Z^n$ has no non-trivial solutions.

Then we have an unbounded sequence of different propositions proven true by the Wiles proof of FLT, q(3), q(4) .....

We have noted in the explanation of this that FLT is not really a well posed theorem because FLT ≡ q(3)^q(4)^ .... and the right hand side of this equivalence lacks meaning in the finite universe of classical logic. This is more to do with usage than anything else. We are just using the natural numbers to list an unbounded sequence of distinct propositions. The structure is essentially:

Wiles proof → q(n) (true) for any nominated natural number n.

The above discussion about GC has the structure:

Goldbach conjecture → p(n) (true) for any nominated natural number $n > 2$.

On the face of it there is a similar structure here, and if Goldbach's conjecture were proved in the conventional sense we would consider the structures identical.

Why then are we able to conclude that GC is unprovable?

The critical difference is that FLT as stated is not a theorem but merely a loose analytic equivalence FLT ≡ q(3)^q(4)^ .....

It is a constructed tautology, albeit meaningless in classical logic. There is no generation of an unbounded number of propositions flowing from the proof of a theorem because the assumed theorem doesn't exist.

The case with GC is quite different.

We have p(2) → p(3) → p(4) → .....

What we do have is a well-defined theorem p(2) which is (finitely) comprehensible, like the Riemann hypothesis, and it is this existence of the non–analytical finite theorem which is important. We have argued in Braun [1], [2], [3] that proof of such a theorem cannot generate an unbounded number of essentially distinct theorems.

We have used the inductive method to uncover p(2) unprovable.

The gap is to establish that p(3), p(4) … are essentially different, in order to appeal to FPFTA.

Intuitively, there would not seem to be any likelihood of establishing a logical relationship between the propositions.

Notes: (July 2019) In the light of later notes focussing on the differences between rational arithmetic and complex analysis as separate systems, this later section needs to clarified and rewritten. We state the obvious here that the Wiles proof of FMT uses objects which have no exact interface with rational arithmetic and so the proof is outside the assumptions of rational arithmetic.

The 'FPFTA' idea seems helpful as an intuitive way of uncovering propositions which may not have proof in rational arithmetic.

[1] P. B. Braun, http://www.peterbraun.com.au/newer%20pdf's/RH.pdf

[2] P. B. Braun, http://www.peterbraun.com.au/newer%20pdf's/29May%20onwards/NEW_DRAFT_16_MAY_2011.pdf

[3] P.B. Braun, http://www.peterbraun.com.au/newer%20pdf's/29May%20onwards/RH_DRAFT%203%20(2).pdf

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